Edexcel Maths FP3

Topic Questions from Papers

Hyperbolic Functions

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1. Solve the equation

$$7\text{sech}\,x - \tanh x = 5$$

Give your answers in the form $\ln a$ where a is a rational number.

(5)

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3. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

(b) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(5)

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5. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

(b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2}\ln k$, where k is an integer.

(5)

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Question 5 continued

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7. $$f(x) = 5\cosh x - 4\sinh x, \qquad x \in \mathbb{R}$$

(a) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$ **(2)**

Hence

(b) solve $f(x) = 5$ **(4)**

(c) show that $\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\,dx = \frac{\pi}{18}$ **(5)**

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Question 7 continued

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7.

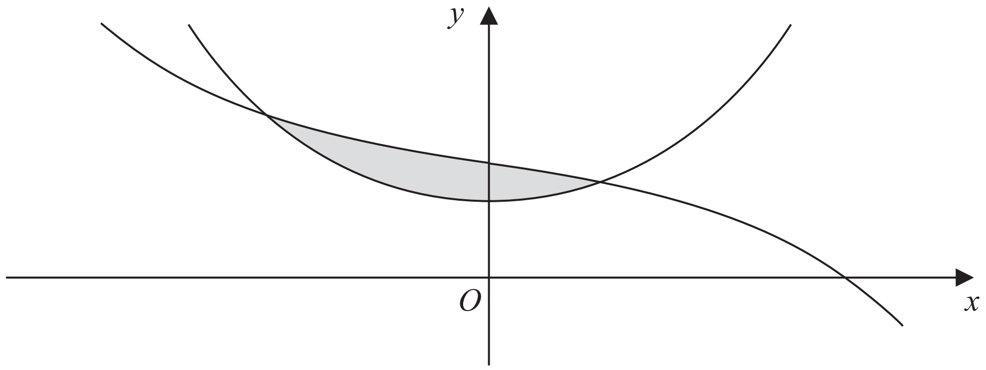

Figure 1

The curves shown in Figure 1 have equations

$$y = 6 \cosh x \quad \text{and} \quad y = 9 - 2 \sinh x$$

(a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x, find exact values for the x-coordinates of the two points where the curves intersect.

(6)

The finite region between the two curves is shown shaded in Figure 1.

(b) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a, b and c are integers.

(6)

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Question 7 continued

Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a.b}}{|\mathbf{b}|}$

The point dividing *AB* in the ratio $\lambda : \mu$ is $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$$\mathbf{a.}(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b.}(\mathbf{c}\times\mathbf{a}) = \mathbf{c.}(\mathbf{a}\times\mathbf{b})$$

If *A* is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through *A* with direction vector **b** has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3}\ (= \lambda)$$

The plane through *A* with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$$n_1x + n_2y + n_3z + d = 0 \text{ where } d = -\mathbf{a.n}$$

The plane through non-collinear points *A*, *B* and *C* has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation

$$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$.

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\text{arcosh } x = \ln\{x + \sqrt{x^2 - 1}\} \qquad (x \geq 1)$$

$$\text{arsinh } x = \ln\{x + \sqrt{x^2 + 1}\}$$

$$\text{artanh } x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Differentiation

f(*x*)	**f′(*x*)**
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\dfrac{1}{1-x^2}$

***Integration* (+ *constant*; $a > 0$ *where relevant*)**

f(*x*)	$\int \mathbf{f}(x)\ \mathbf{d}x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\dfrac{x}{a}\right),\quad \ln\left\{x+\sqrt{x^2-a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\dfrac{x}{a}\right),\quad \ln\left\{x+\sqrt{x^2+a^2}\right\}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\operatorname{artanh}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Arc length

$$s = \int \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, \mathrm{d}x \qquad \text{(cartesian coordinates)}$$

$$s = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\, \mathrm{d}t \qquad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \;\mathrm{d}s = 2\pi \int y \sqrt{\left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\, \mathrm{d}x$$

$$= 2\pi \int y \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\, \mathrm{d}t$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathbf{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert,\quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert,\quad \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B \equiv 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B \equiv 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B \equiv 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B \equiv -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	**f′(x)**
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$